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SCIENCE TECHNOLOGY ENGINEERING MATHEMATICS

Appunti
Sistemi Trifase

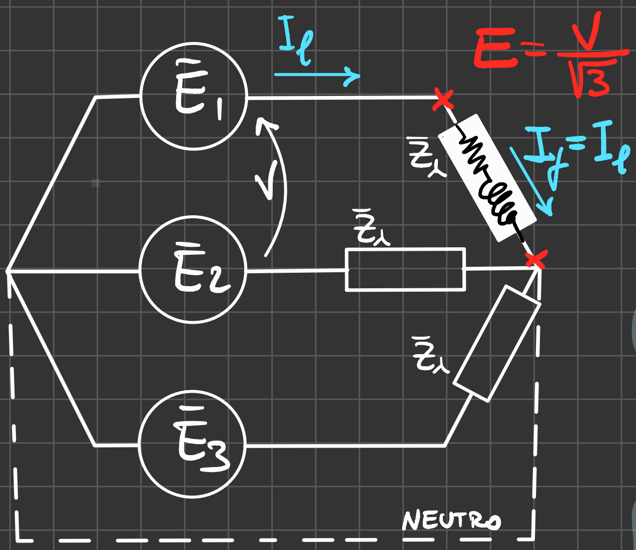
IIS2 - ELETTROTECNICA

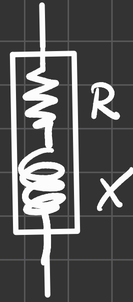
rev.0.9 - 02 set 2023

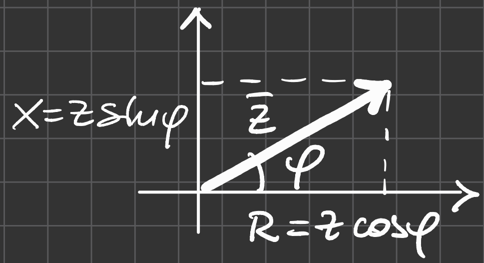
Draft version

Appunti in formato bozza, intesi esclusivamente di ausilio alle lezioni, che le integrano nelle descrizioni e nei ragionamenti su quanto viene riportato in queste pagine.

POTENZA λ







$$\vec{Z}_1 = \vec{Z}_2 = \vec{Z}_3 = \vec{Z}$$

$$\vec{Z} = (R + jX)$$

$$I_z = I_f = I_\ell \qquad V_z = E$$

$$\begin{cases} Z = \sqrt{R^2 + X^2} \\ \varphi = \operatorname{arctg} \frac{X}{R} \end{cases}$$

$\vec{E} = \vec{I}_\ell \vec{Z}_\lambda$ ohm

$$|Z_\lambda| = \frac{E}{I_\ell} = \frac{V}{\sqrt{3} I_\ell} < \begin{matrix} R = Z_\lambda \cos\varphi \\ X = Z_\lambda \sin\varphi \end{matrix}$$

$$\bar{Z} = Z(\cos\varphi + j\sin\varphi)$$

$E_1 = E_2 = E_3 = E \qquad \bar{E}_1 = E\angle 0 \qquad \bar{E}_2 = E\angle{-120} \qquad \bar{E}_3 = E\angle{-240} = E\angle 120$

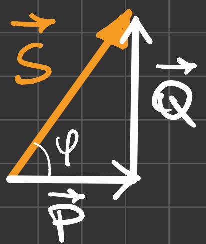
$V = \sqrt{3}E$ 30° in anticipo $\quad V_{12} = V\angle 30° \qquad V_{23} = V\angle{-90°} \qquad V_{31} = V\angle 150°$

$$P_{R_1} = V_{R_1} \underset{I_\ell}{I_{R_1}} = I_\ell^2 \cdot R \xrightarrow[P_{TOT} = P_{R_1} + P_{R_2} + P_{R_3}]{\text{Teorema BOUCHEROT}} \boxed{P = 3 I_\ell^2 \cdot R}$$

$$Q_{X_1} = V_{X_1} \underset{I_\ell}{I_{X_1}} = I_\ell^2 \cdot X \xrightarrow[Q_{TOT} = Q_{X_1} + Q_{X_2} + Q_{X_3}]{\text{Teorema BOUCHEROT}} \boxed{Q = 3 I_\ell^2 \cdot X}$$

$$S_{Z_1} = \underset{E}{V_{Z_1}} \underset{I_\ell}{I_{Z_1}} = E I_\ell = I_\ell^2 Z \xrightarrow[S_{TOT} = S_{Z_1} + S_{Z_2} + S_{Z_3}]{\text{Teorema BOUCHEROT}} \boxed{S = 3 \underset{V/\sqrt{3}}{E} I_\ell = \sqrt{3} V I_\ell}$$

$$= 3 I_\ell^2 Z$$



$$P = S\cos\varphi = 3 \underset{V/\sqrt{3}}{E} I_\ell \cos\varphi = \sqrt{3} V I_\ell \cos\varphi$$

$$= 3 I_\ell^2 \underbrace{Z\cos\varphi}_{R} \Rightarrow R = \frac{P}{3 I_\ell^2}$$

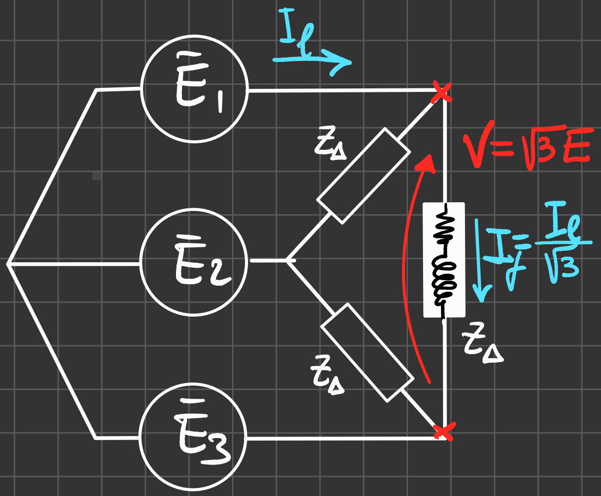
$$S = \sqrt{P^2 + Q^2}$$

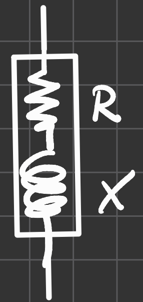
$$\varphi = \operatorname{arctg} \frac{X}{R}$$

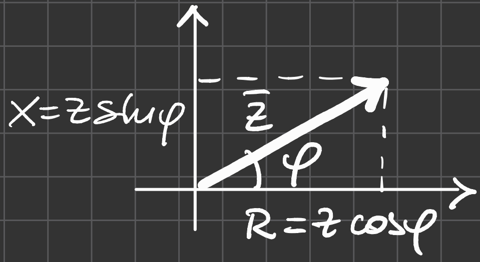
$$Q = S\sin\varphi = 3 E I_\ell \sin\varphi = \sqrt{3} V I \sin\varphi$$

$$= 3 I_\ell^2 \underbrace{Z\sin\varphi}_{X} \Rightarrow X = \frac{Q}{3 I^2}$$

POTENZA Δ







$\vec{Z}_1=\vec{Z}_2=\vec{Z}_3=\vec{Z}$

$I_z=I_f=\frac{I_\ell}{\sqrt{3}}$ $\quad V_z=\sqrt{3}E$

$\vec{V}=\vec{I}_f\vec{Z}_\Delta$ ohm

$\vec{Z}=(R+jX)$

$\begin{cases} z=\sqrt{R^2+X^2} \\ \varphi=\operatorname{arctg}\frac{X}{R} \end{cases}$

$\bar{z}=z(\cos\varphi+j\sin\varphi)$

$\boxed{Z_\Delta=3Z_\lambda}$ $\leftarrow$ $|Z|_\Delta=\frac{\sqrt{3}E}{\frac{I_\ell}{\sqrt{3}}}=\frac{3E}{I_\ell}=$ $\begin{cases} R=z_\Delta\cos\varphi \\ X=z_\Delta\sin\varphi \end{cases}$

$E_1=E_2=E_3=E$ $\quad \bar{E}_1=E\angle 0$ $\quad \bar{E}_2=E\angle -120$ $\quad \bar{E}_3=E\angle -240=E\angle 120$

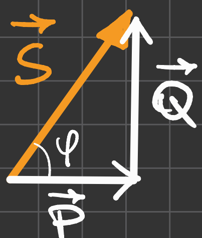
$V=\sqrt{3}E$ 30° in anticipo $\quad V_{12}=V\angle 30°$ $\quad V_{23}=V\angle -90°$ $\quad V_{31}=V\angle 150°$

$P_{R_1}=V_{R_1}\underbrace{I_{R_1}}_{I_f=\frac{I_\ell}{\sqrt{3}}}=I_f^2\cdot R=\frac{I_\ell^2}{3}\cdot R \xrightarrow[P_{TOT}=P_{R_1}+P_{R_2}+P_{R_3}]{\text{Teorema BOUCHEROT}} P=\cancel{3}\frac{I_\ell^2}{\cancel{3}}\cdot R=3I_f^2R$ ⚠ I_ℓ

$Q_{X_1}=V_{X_1}\underbrace{I_{X_1}}_{I_f=\frac{I_\ell}{\sqrt{3}}}=I_f^2\cdot X=\frac{I_\ell^2}{3}\cdot X \xrightarrow[Q_{TOT}=Q_{X_1}+Q_{X_2}+Q_{X_3}]{\text{Teorema BOUCHEROT}} Q=\cancel{3}\frac{I_\ell^2}{\cancel{3}}\cdot X=3I_f^2X$

$S_{z_1}=\underbrace{V_{z_1}}_{\sqrt{3}E}\underbrace{I_{z_1}}_{I_f=\frac{I_\ell}{\sqrt{3}}}=EI_\ell=I_\ell^2 Z \xrightarrow[S_{TOT}=S_{z_1}+S_{z_2}+S_{z_3}]{\text{Teorema BOUCHEROT}} S=3\underbrace{E}_{\frac{V}{\sqrt{3}}}I_\ell=\sqrt{3}VI_\ell$

$=3I_\ell^2Z$



$S=\sqrt{P^2+Q^2}$

$\varphi=\operatorname{arctg}\frac{X}{R}$

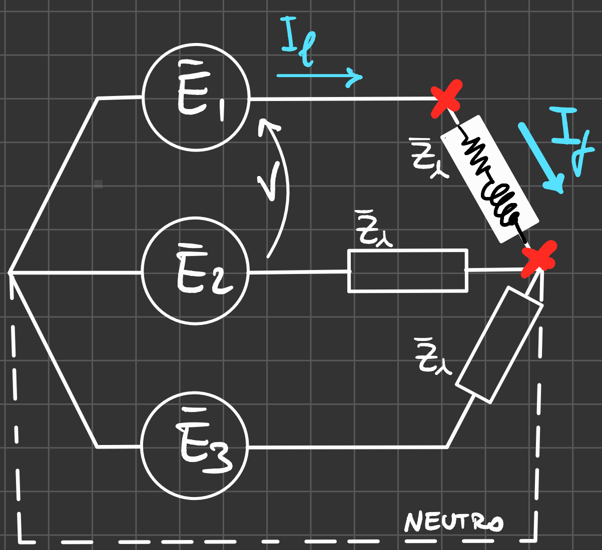
$P=S\cos\varphi=3\underbrace{V_z}_{V=\sqrt{3}E}\underbrace{I_z}_{I_f=\frac{I_\ell}{\sqrt{3}}}\cos\varphi=3EI_\ell\cos\varphi=\sqrt{3}VI_\ell\cos\varphi$

$=3I_f^2\underbrace{z\cos\varphi}_{R}\Rightarrow R=\frac{P}{3I_f^2}=\frac{P}{3\left(\frac{I_\ell}{\sqrt{3}}\right)^2}$

$Q=S\sin\varphi=3\underbrace{V_z}_{V=\sqrt{3}E}\underbrace{I_z}_{I_f=\frac{I_\ell}{\sqrt{3}}}\cos\varphi=3EI_\ell\sin\varphi=\sqrt{3}VI_\ell\sin\varphi$

$=3I_f^2\underbrace{z\sin\varphi}_{X}\Rightarrow X=\frac{Q}{3I_f^2}=\frac{Q}{3\left(\frac{I_\ell}{\sqrt{3}}\right)^2}$

CARICO EQUILIBRATO λ



$$V_{Z_\lambda} = E = \frac{V}{\sqrt{3}}$$

$$I_{Z_\lambda} = I_f = I_\ell$$

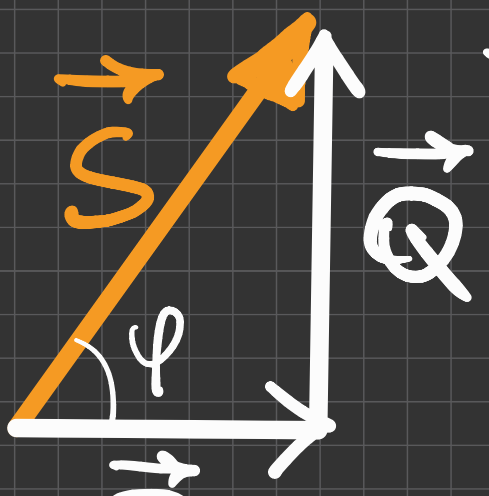
stesso corrente in tutti i componenti

APPARENTE $S = 3 \cdot V_{Z_\lambda} I_{Z_\lambda} = 3 \cdot E I_\ell = 3 I_\ell^2 \cdot Z_\lambda$ [VA]

ATTIVA $P = 3 \cdot V_{R_\lambda} I_{R_\lambda} = 3 I_{R_\lambda}^2 \cdot R_\lambda = 3 I_\ell^2 \cdot R_\lambda$ [W]

REATTIVA $Q = 3 \cdot V_{X_\lambda} I_{X_\lambda} = 3 I_{X_\lambda}^2 \cdot X_\lambda = 3 I_\ell^2 \cdot X_\lambda$ [VAR]

$$S = \sqrt{P^2 + Q^2} \qquad \varphi = \operatorname{arctg} \frac{X}{R}$$



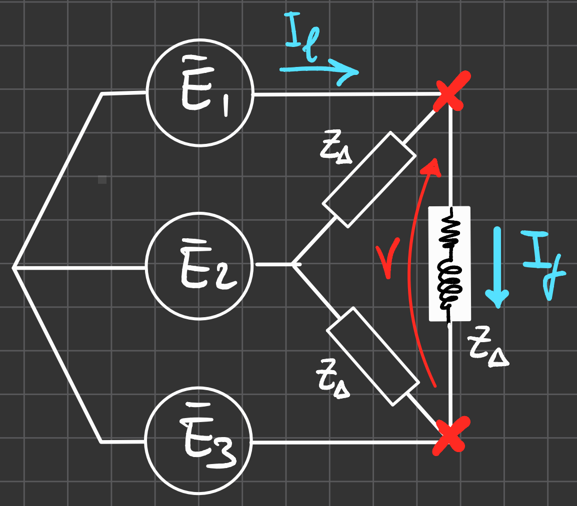
$$\vec{Q} = S \sin\varphi = 3 E I_\ell \sin\varphi = \sqrt{3} V I_\ell \sin\varphi$$

$$= 3 I_\ell^2 \underbrace{Z \sin\varphi}_{X} \Rightarrow X = \frac{Q}{3 I_\ell^2}$$

$$\vec{P} = S \cos\varphi = 3 \overset{\frac{V}{\sqrt{3}}}{E} I_\ell \cos\varphi = \sqrt{3} V I_\ell \cos\varphi$$

$$= 3 I_\ell^2 \underbrace{Z \cos\varphi}_{R} \Rightarrow R = \frac{P}{3 I_\ell^2}$$

CARICO EQUILIBRATO Δ



$$V_{Z_\Delta} = V = \sqrt{3}E$$

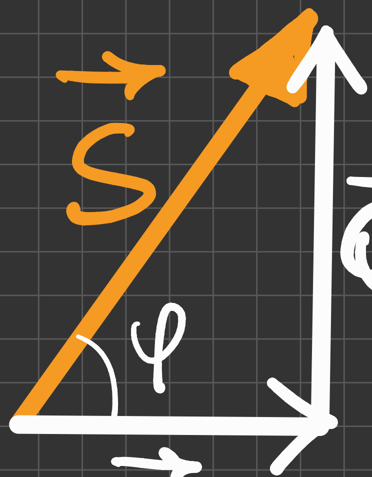
$$I_{Z_\Delta} = I_f = \frac{I_l}{\sqrt{3}}$$

APPARENTE $S = 3\cdot V_{Z_\Delta} I_{Z_\Delta} = 3\cdot\cancel{\sqrt{3}}E\frac{I_l}{\cancel{\sqrt{3}}} = 3I_l^2\cdot Z_\Delta$ [VA]

ATTIVA $P = 3\cdot V_{R_\Delta} I_{R_\Delta} = 3I_R^2\cdot R_\Delta = \cancel{3}\left(\frac{I_l}{\cancel{\sqrt{3}}}\right)^2 R_\Delta = I_l^2\cdot R_\Delta$ [W]

REATTIVA $Q = 3\cdot V_{X_\Delta} I_{X_\Delta} = 3I_{X_\Delta}^2\cdot X_\Delta = \cancel{3}\left(\frac{I_l}{\cancel{\sqrt{3}}}\right)^2 R_\Delta = I_l^2\cdot X_\Delta$ [VAR]

$$S = \sqrt{P^2+Q^2} \qquad \varphi = \mathrm{arctg}\,\frac{X}{R}$$



$$\vec{Q} = S\sin\varphi = 3\overset{V=\sqrt{3}E}{V_z}\overset{I_f=\frac{I_l}{\sqrt{3}}}{I_z}\cos\varphi = 3EI_l\sin\varphi = \sqrt{3}VI_l\sin\varphi$$

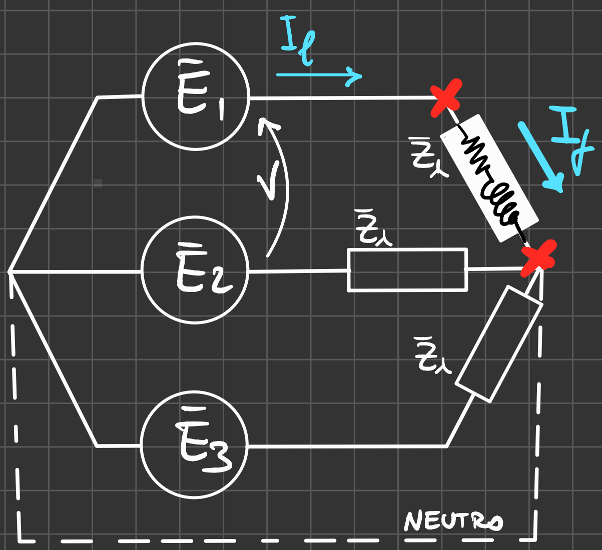
$$= 3I_f^2\underbrace{z\sin\varphi}_{X} \Rightarrow X = \frac{Q}{3I_f^2} = \frac{Q}{\cancel{3}\left(\frac{I_l}{\cancel{\sqrt{3}}}\right)^2}$$

$$\vec{P} = S\cos\varphi = 3\overset{V=\sqrt{3}E}{V_z}\overset{I_f=\frac{I_l}{\sqrt{3}}}{I_z}\cos\varphi = 3EI_l\cos\varphi = \sqrt{3}VI_l\cos\varphi$$

$$= 3I_f^2\underbrace{z\cos\varphi}_{R} \Rightarrow R = \frac{P}{3I_f^2} = \frac{P}{\cancel{3}\left(\frac{I_l}{\cancel{\sqrt{3}}}\right)^2}$$

confronto:

CARICO EQUILIBRATO $\curlywedge$

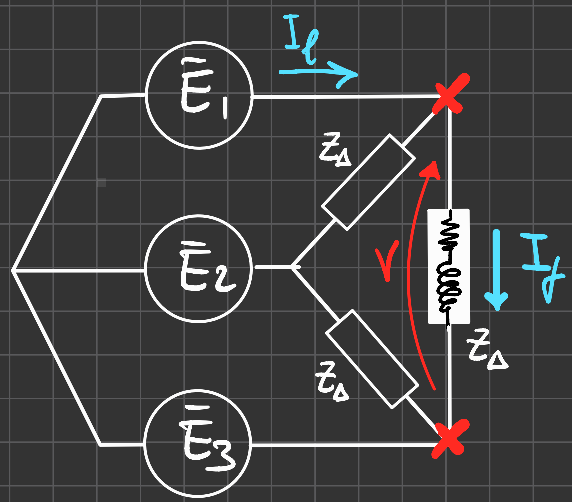


$$V_{Z_\lambda} = E = \frac{V}{\sqrt{3}}$$

$$I_{Z_\lambda} = I_f = I_\ell$$

$$R = \frac{P}{3 \cdot I_\ell^2} \; ; \; X = \frac{Q}{3 \cdot I_\ell^2}$$

CARICO EQUILIBRATO Δ



$$V_{Z_\Delta} = V = \sqrt{3} E$$

$$I_{Z_\Delta} = I_f = \frac{I_\ell}{\sqrt{3}}$$

$$R = \frac{P}{I_\ell^2} \; ; \; X = \frac{Q}{I_\ell^2}$$

$$S = 3 E I_\ell = \sqrt{3} V I_\ell$$

$$S = \sqrt{P^2 + Q^2}$$

$$\varphi = \operatorname{arctg} \frac{Q}{P} = \operatorname{arctg} \frac{X}{R}$$

$$Q = S \sin\varphi$$

$$P = S \cos\varphi$$

$$R_\Delta = 3 R_\lambda$$

$$X_\Delta = 3 X_\lambda$$

$$Z_\Delta = 3 Z_\lambda$$

5. Un carico equilibrato collegato a stella è alimentato con tensione concatenata $V = 400\ \mathrm{V}$. Esso dissipa una potenza $P = 2\ \mathrm{kW}$ e assorbe una corrente di linea $I_l = 4\ \mathrm{A}$. Determina la resistenza e la reattanza del carico e inoltre il nuovo valore di P se le stesse impedenze venissero collegate a triangolo.

Unità 5 Potenze di sistemi trifasi

Esercizi di fine unità

2. $I = 144\ \mathrm{A}$; $R_{\curlywedge} = 1{,}286\ \Omega$; $X_{\curlywedge} = 0{,}964\ \Omega$; $R_{\Delta} = 3{,}858\ \Omega$; $X_{\Delta} = 2{,}892\ \Omega$

5. $R_{\curlywedge} = 41{,}6\ \Omega$; $X_{\curlywedge} = 39{,}95\ \Omega$; $P_{\Delta} = 6\ \mathrm{kW}$

6. $Z_{\Delta} = (120 - j60)\ \Omega$

7. $\overline{Z}_{\curlywedge} = (28.8 + j14.1)\ \Omega$

9. $P_T = 14{,}4\ \mathrm{kW}$; $Q_T = 5{,}86\ \mathrm{kVAR}$

10. $\overline{I}_1 = 6.95\ \mathrm{A}\ \angle 32°$; $\overline{I}_2 = 14.5\ \mathrm{A}\ \angle -101°$; $\overline{I}_3 = 11\ \mathrm{A}\ \angle 106°$; $P_T = 7.05\ \mathrm{kW}$; $Q_T = -1.27\ \mathrm{kVAR}$; $I_{\curlywedge} = 4.62\ \mathrm{A}$; $I_{12} = I_{23} = 8\ \mathrm{A}$

12. $C = 67.9\ \mu\mathrm{F}$; $I_{l_1} = 28.8\ \mathrm{A}$; $I_{l_2} = 19.2\ \mathrm{A}$

13. $C = 3{,}66\ \mu\mathrm{F}$; $P = 1377\ \mathrm{W}$; $I_l' = 2{,}14\ \mathrm{A}$; $I_l'' = 1{,}98\ \mathrm{A}$

14. $C = 3{,}4\ \mu\mathrm{F}$; $P = 1000\ \mathrm{W}$; $I_l' = 2{,}04\ \mathrm{A}$; $I_l'' = 1{,}6\ \mathrm{A}$

15. $Q = 1053\ \mathrm{VAR}$

Verifica di fine unità

1. C **2.** D **3.** C **4.** B **5.** B **6.** D **7.** A

1. Collegamento a stella

$$S = \sqrt{P^2 + Q^2} = 2{,}236\ \mathrm{kVA}$$

$$I_l = \frac{S}{\sqrt{3} \cdot V} = \frac{2236}{660} = 3{,}4\ \mathrm{A}$$

$$R_{\curlywedge} = \frac{P}{3I^2} = \frac{1000}{3 \cdot 3{,}4^2} = 28{,}8\ \Omega$$

$$X_{\curlywedge} = \frac{Q}{3I^2} = \frac{2000}{3 \cdot 3{,}4^2} = 57{,}7\ \Omega$$

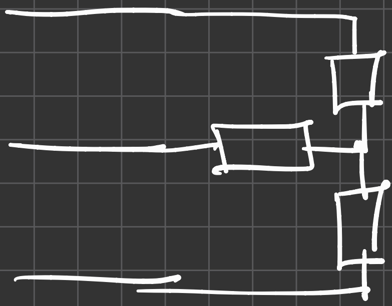
2. Collegamento a triangolo

La corrente di linea rimane costante mentre la corrente di fase risulta:

$$I_F = \frac{I_l}{\sqrt{3}} = 1{,}96\ \mathrm{A};\ R_{\Delta} = \frac{P}{3I_F^2} = 86{,}6\ \Omega;$$

$$X_{\Delta} = \frac{Q}{3I_F^2} = 173{,}3\ \Omega$$

Si osservi che $\overline{Z}_{\Delta} = 3\overline{Z}_{\curlywedge}$, a meno delle approssimazioni di calcolo.



$V = 400\,\mathrm{V}$

$P = 2\,\mathrm{kW}$

$I_e = 4\,\mathrm{A}$

$$P = \sqrt{3}\, V I \cos\varphi$$

$$\cos\varphi = \frac{P}{\sqrt{3}\, V I} = \frac{\cancel{2} \cdot 10^{\cancel{3}}}{\sqrt{3} \cdot 4 \cdot \cancel{10^2} \cdot \cancel{4}_2} = \frac{10}{\sqrt{3} \cdot 8} = 0{,}72$$

$$\varphi = \arccos 0{,}72 = 44°$$

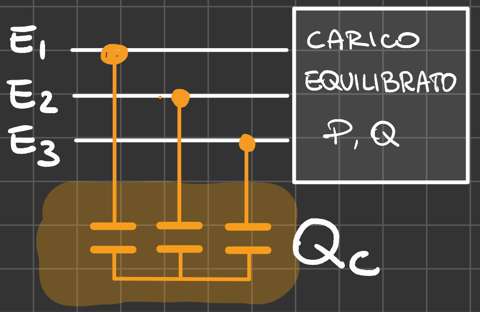
$$1)\ R = \frac{P}{3I^2} = \frac{\cancel{2} \cdot 10^3}{3 \cdot 4 \cdot \cancel{4}2} = \frac{10^3}{24} = 41{,}67\ \Omega$$

$$S = \sqrt{3}\, V I = \sqrt{3} \cdot 400 \cdot 4 = 2771\ \mathrm{W}$$

$$Q = \sqrt{S^2 - P^2} = \sqrt{2771^2 - 2000^2} = 1918\ \mathrm{VAR}$$

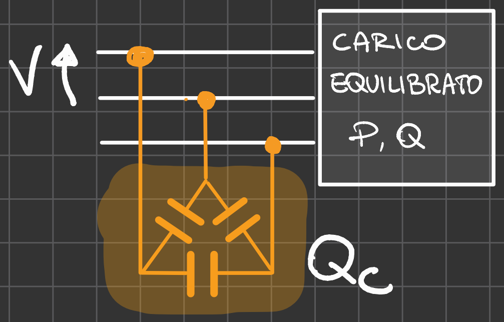
$$2)\ X = \frac{Q}{3I^2} = \frac{1918}{3 \cdot 4 \cdot 4} = 39{,}95$$

SISTEMI TRIFASE: RIFASAMENTO



collegamento STELLA
∀ condensatore:

$Q_{C\lambda} = \frac{1}{3} Q_c \qquad V_{C\lambda} = E$

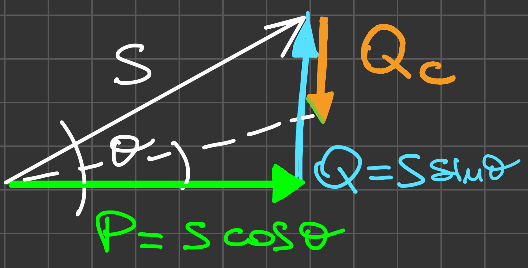


collegamento Triangolo
∀ condensatore:

$Q_{C\Delta} = \frac{1}{3} Q_c \qquad V_{C\Delta} = V = \sqrt{3} E$

Il rifasamento serve a compensare una potenza reattiva induttiva rilevante (comportamento ohmico-induttivo), ovvero per diminuire lo sfasamento φ tra tensione e corrente. Per convenzione assumiamo:

$$\cos\varphi_{max} \simeq 0{,}9 \rightarrow \varphi_{max} = 26° \rightarrow tg\,\varphi_{max} \simeq 0{,}48$$



$$S = \sqrt{P^2+Q^2}$$

$$\cos\varphi = \frac{P}{S}; \quad \sin\varphi = \frac{Q}{S}; \quad tg\varphi = \frac{Q}{P}$$

calcoliamo Q_c:

$$Q_{TOT} = Q - Q_c \rightarrow Q_c = \overset{P\,tg\varphi}{Q} - \overset{P\,tg\varphi_{max}}{Q_{TOT}} = P(tg\varphi - tg\varphi_{max})$$

calcoliamo C_λ: $\overset{\frac{Q_c}{\cancel{3}}}{Q_{C\lambda}} = \frac{V_{C\lambda}^2}{X_{C\lambda}}$ $\quad E^2 = \left(\frac{V}{\sqrt{3}}\right)^2 = \frac{V^2}{\cancel{3}}$ $\rightarrow \overset{\frac{1}{\omega C_\lambda}}{X_{C\lambda}} = \frac{V^2}{Q_c} \rightarrow C_\lambda = \frac{Q_c}{\omega V^2}$

calcoliamo C_λ: $\overset{\frac{Q_c}{3}}{Q_{C\Delta}} = \frac{\overset{V^2}{V_{C\Delta}^2}}{X_{C\Delta}} \rightarrow \overset{\frac{1}{\omega C_\Delta}}{X_{C\Delta}} = \frac{3V^2}{Q_{C\Delta}} \rightarrow C_\Delta = \frac{Q_c}{3\omega V^2}$