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SCIENCE TECHNOLOGY ENGINEERING MATHEMATICS

# Appunti
# Filtri

## Passa Basso, Passa Alto, Passa Banda

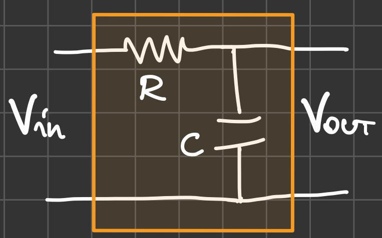
IIS2 - ELETTROTECNICA

rev.0.9 - 02 set 2023

Draft version

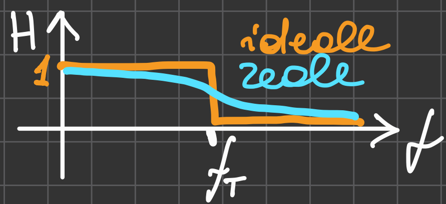
Appunti in formato bozza, intesi esclusivamente di ausilio alle lezioni, che le integrano nelle descrizioni e nei ragionamenti su quanto viene riportato in queste pagine.

# FILTRO PASSA BASSO




Funzione di Trasferimento

$$\bar H = \frac{\bar V_{OUT}}{\bar V_{in}}$$




partitore di tensione $\bar V_{OUT} = \frac{\bar V_{in}\cdot \bar Z_c}{\bar Z_c + \bar Z_R} \rightarrow \bar H = \frac{\bar Z_c}{\bar Z_c + \bar Z_R}$

con $\bar Z_R = (R + j\emptyset)$ $\bar Z_c = (0 - jX_c)$ $\frac{1}{wC}$

$$\bar H = \frac{\frac{-j}{wC}}{R - \frac{j}{wC}} \cdot \frac{jwC}{jwC} = \frac{1}{1 + jwRC}$$

**modulo :** modulo numeratore / modulo denominatore

$$H = \frac{1}{\sqrt{1+(wRC)^2}} =$$

per $f \rightarrow 0$, $(wRC)^2 \ll 1$ $\longrightarrow H = \frac{1}{\sqrt{1+\cancel{(wRC)^2}}} = 1$ ← molto piccolo

per $f \rightarrow \infty$, $(wRC)^2 \gg 1$ $\longrightarrow H = \frac{1}{\sqrt{\cancel{1}+(wRC)^2}} = 0$ ← molto grande

**FASE :** FAse numeratore – Fase denominatore

$$\angle H = \operatorname{arctg}\frac{0}{1} - \operatorname{arctg}\frac{wRC}{1} = -\operatorname{arctg}(wRC)$$

per $f \rightarrow 0$, $(wRC) \rightarrow 0$ $\longrightarrow \angle H = 0^\circ$

per $f \rightarrow \infty$, $(wRC) \rightarrow \infty$ $\longrightarrow \angle H = -90^\circ$

**Frequenza di taglio**: Si ricava imponendo una caduta di segnale di 3 db, corrispondente a $\frac{1}{\sqrt 2}$

$$H = \frac{1}{\sqrt 2} \rightarrow \frac{1}{\sqrt{\underbrace{1+(wRC)^2}_{1}}} = \frac{1}{\sqrt 2} \rightarrow (wRC)^2 = 1 \rightarrow \boxed{W_T = \frac{1}{RC} \quad f_T = \frac{W_T}{2\pi} = \frac{1}{2\pi RC}}$$

per $w = W_T$ $\angle H = -\operatorname{arctg}\left(\frac{1}{RC}RC\right) = -\operatorname{arctg} 1 = -45^\circ$

sostituendo $RC = \frac{1}{\omega_T}$

$$H = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_T}\right)^2}}$$

$$\angle H = -\operatorname{arctg}\frac{\omega}{\omega_T}$$

# RIASSUMENDO:

$$H = \frac{1}{1+j\omega RC}$$

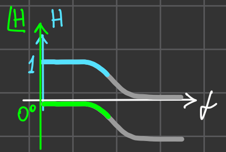
$$H = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_T}\right)^2}} = \frac{1}{\sqrt{1+\left(\frac{f}{f_T}\right)^2}}$$

$$\angle H = -\operatorname{arctg}\frac{\omega}{\omega_T} = -\operatorname{arctg}\left(\frac{f}{f_T}\right)$$

con $\omega_T = 2\pi f_T = \frac{1}{RC}$

per $f \to 0$

$(\omega RC) \to 0$

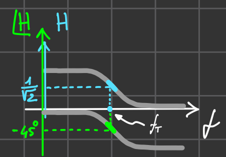



$$H = \frac{1}{\sqrt{1+\cancel{(\omega RC)^2}}} = 1$$

$$\angle H = -\operatorname{arctg}(\omega RC) = 0°$$

per $f = f_T$

$\omega = \omega_T$

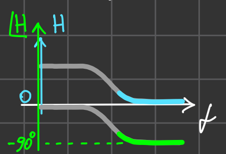



$$H = \frac{1}{\sqrt{1+\left(\frac{\cancel{RC}}{\cancel{RC}}\right)^2}} = \frac{1}{\sqrt{2}}$$

$$\angle H = -\operatorname{arctg}\frac{\cancel{RC}}{\cancel{RC}} = -45°$$

per $f \to \infty$

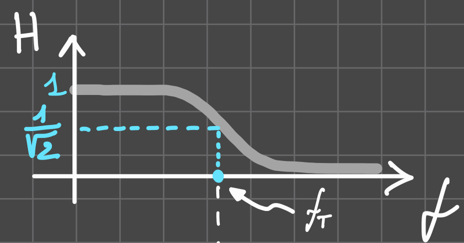
$(\omega RC) \to \infty$



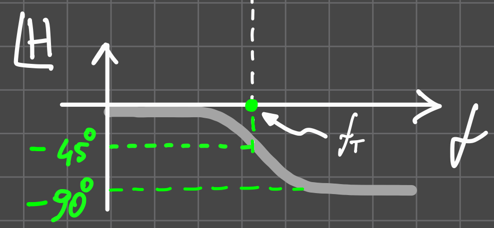

$$H = \frac{1}{\sqrt{\cancel{1}+(\omega RC)^2}} = 0$$

$$\angle H = -\operatorname{arctg}(\omega RC) = -90°$$

## modulo




## FASE



# DIMENSIONAMENTO di un filtro passa basso

Assegno $f_T \xrightarrow{\omega_T = 2\pi f = \frac{1}{RC}}$ ricavo i valori di R e C

$$
\begin{array}{lllll}
 & \nearrow \text{Hz} & \rightarrow C & mF & \searrow \\
f_T & \rightarrow \text{kHz} & \rightarrow C & \mu F & \rightarrow R \text{ di conseguenza} \\
 & \searrow \text{MHz} & \rightarrow C & nF & \nearrow
\end{array}
$$

Esempio:

Progettare un filtro RC passa basso con $f_T = 3$ kHz

$$f_T = 18\,kHz \rightarrow \omega_T = 2\pi\cdot 3\cdot 10^3 = 1{,}88\cdot 10^4 \ \frac{rad}{sec}$$

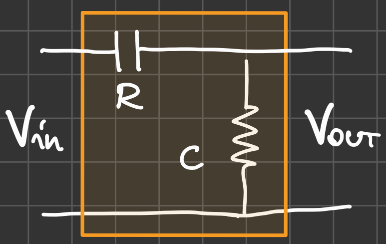
$$\omega_T = \frac{1}{RC} \rightarrow RC = \frac{1}{\omega_T} = \frac{1}{1{,}88\cdot 10^5} = \underbrace{5{,}32\cdot 10^{-5}\cdot 10^{-2}}_{C}\cdot\underbrace{10^2}_{R}$$

$C = 0{,}532\ \mu F$

$R = 100\ \Omega$

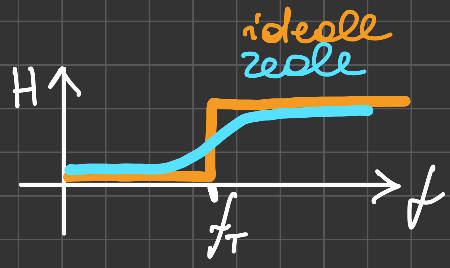
NB la scelta di dimensionare C dell'ordine dei $\mu F$ deriva dalla richiesta di $f_T$ in kHz

# FILTRO PASSA ALTO




Funzione di Trasferimento

$$\bar H = \frac{\bar V_{out}}{\bar V_{in}}$$




partitore di tensione $\bar V_{out} = \frac{\bar V_{in} \cdot \bar Z_R}{\bar Z_C + \bar Z_R} \rightarrow \bar H = \frac{\bar Z_R}{\bar Z_C + \bar Z_R}$

con $\bar Z_R = (R + j\emptyset)$ $\bar Z_C = (\emptyset - j X_C)$ $\frac{1}{wC}$

$$\bar H = \frac{R}{R - \frac{j}{wC}} \cdot \frac{jwC}{jwC} = \frac{jwRC}{1 + jwRC} = \frac{1}{1 + \frac{1}{jwRC} \cdot \frac{j}{j}} = \frac{1}{1 - j\frac{1}{wRC}}$$

**modulo :** modulo numeratore / modulo denominatore

$$H = \frac{1}{\sqrt{1 + \frac{1}{(wRC)^2}}} =$$

per $f \rightarrow 0$, $(wRC)^2 \ll 1$ $\rightarrow H = \frac{1}{\sqrt{\cancel{1} + \frac{1}{(wRC)^2}}} = 0$ ← molto grande

per $f \rightarrow \infty$, $(wRC)^2 \gg 1$ $\rightarrow H = \frac{1}{\sqrt{1 + \cancel{\frac{1}{(wRC)^2}}}} = 1$ ← molto piccolo

**FASE :** FAse numeratore − Fase denominatore

$$\angle H = \operatorname{arctg}\frac{0}{1} + \operatorname{arctg}\frac{+\frac{1}{wRC}}{1} = \operatorname{arctg}\left(\frac{1}{wRC}\right)$$

$tg(-\alpha) = -tg\alpha$

per $f \rightarrow 0$, $(1/wRC) \rightarrow \infty$ $\rightarrow \angle H = 90^\circ$

per $f \rightarrow \infty$, $(1/wRC) \rightarrow 0$ $\rightarrow \angle H = 0^\circ$

**Frequenza di taglio :** Si ricava imponendo una caduta di segnale di 3 db, corrispondente a $\frac{1}{\sqrt{2}}$

$$H = \frac{1}{\sqrt{2}} \rightarrow \frac{1}{\sqrt{1 + \frac{1}{\underbrace{(wRC)^2}_{1}}}} = \frac{1}{\sqrt{2}} \rightarrow (wRC)^2 = 1 \rightarrow$$

$$\boxed{w_T = \frac{1}{RC} \qquad f_T = \frac{w_T}{2\pi} = \frac{1}{2\pi RC}}$$

per $w = w_T$ $\angle H = \operatorname{arctg}\left(\frac{1}{\frac{1}{RC}RC}\right) = \operatorname{arctg} 1 = 45^\circ$

sostituendo $RC = \frac{1}{\omega_T}$ $\begin{cases} H = \dfrac{1}{\sqrt{1+\left(\frac{\omega_T}{\omega}\right)^2}} \\ \angle H = \operatorname{arctg}\dfrac{\omega_T}{\omega} \end{cases}$

# RIASSUMENDO:

$$H = \frac{1}{1 - j\frac{1}{\omega RC}} \quad \begin{cases} H = \dfrac{1}{\sqrt{1+\left(\frac{\omega_T}{\omega}\right)^2}} = \dfrac{1}{\sqrt{1+\left(\frac{f_T}{f}\right)^2}} \\ \angle H = \operatorname{arctg}\dfrac{\omega_T}{\omega} = \operatorname{arctg}\left(\dfrac{f_T}{f}\right) \end{cases}$$

con $\omega_T = 2\pi f_T = \frac{1}{RC}$

per $f \to 0$
$(\omega RC) \to 0$

$$H = \frac{1}{\sqrt{\cancel{1+}\frac{1}{(\omega RC)^2}}} = 0$$

$$\angle H = \operatorname{arctg}\left(\frac{1}{\omega RC}\right) = 90^\circ$$

per $f = f_T$
$\omega = \omega_T$

$$H = \frac{1}{\sqrt{1+\left(\frac{1}{\frac{1}{\cancel{RC}}\cancel{RC}}\right)^2}} = \frac{1}{\sqrt{2}}$$

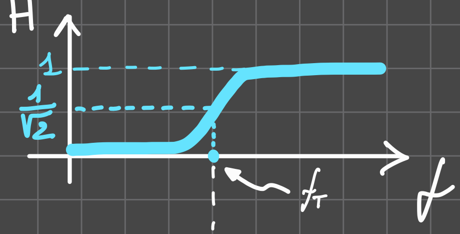
$$\angle H = \operatorname{arctg}\left(\frac{1}{\frac{1}{\cancel{RC}}\cancel{RC}}\right) = 45^\circ$$

per $f \to \infty$
$(\omega RC) \to \infty$

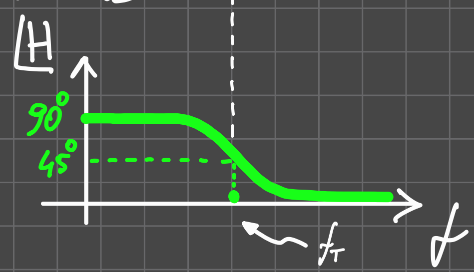
$$H = \frac{1}{\sqrt{1+\cancel{\frac{1}{(\omega RC)^2}}}} = 1$$

$$\angle H = \operatorname{arctg}\left(\frac{1}{\omega RC}\right) = 0^\circ$$

modulo




FASE



## DIMENSIONAMENTO di un filtro passa alto

Assegno $f_T \xrightarrow{\omega_T = 2\pi f = \frac{1}{RC}}$ ricavo i valori di R e C

$$
f_T \begin{cases} Hz \rightarrow C \quad mF \\ kHz \rightarrow C \quad \mu F \\ MHz \rightarrow C \quad nF \end{cases} \Rightarrow R \text{ di conseguenza}
$$

NB procedura analoga al dimensionamento dei filtri RC passa basso

Esempio:

Progettare un filtro RC passa alto con $f_T = 50\ Hz$

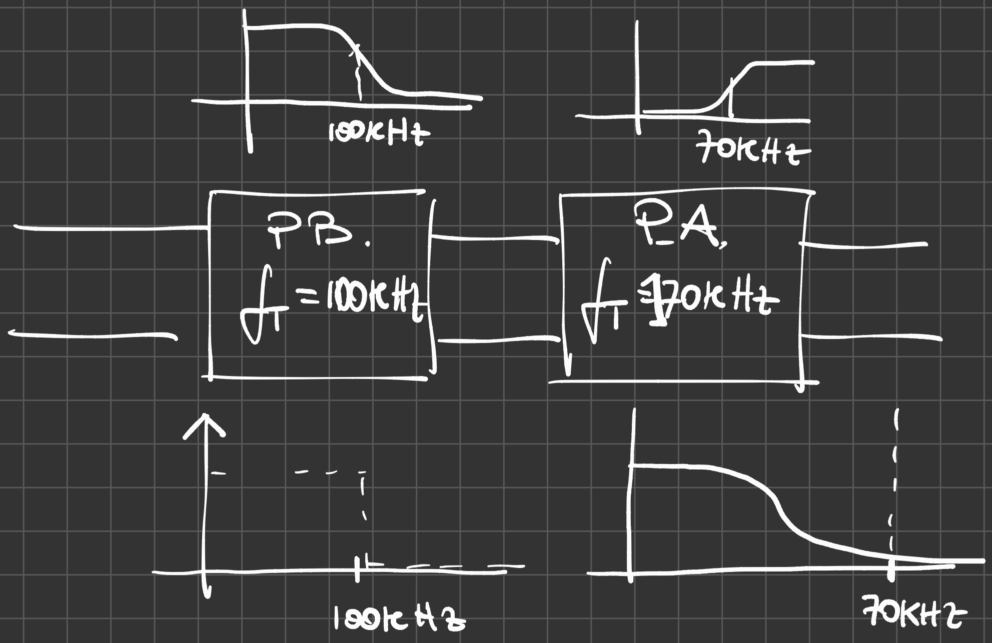
$$f_T = 18\ kHz \rightarrow \omega_T = 2\pi \cdot 50 = 3{,}14 \cdot 10^2\ \frac{rad}{sec}$$

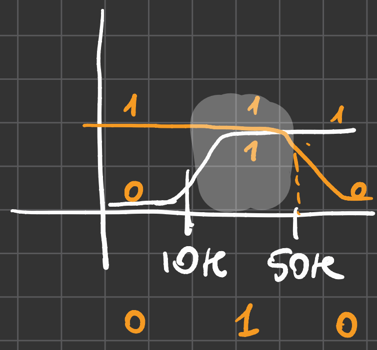
$$\omega_T = \frac{1}{RC} \rightarrow RC = \frac{1}{\omega_T} = \frac{1}{3{,}14 \cdot 10^2} = \underbrace{0{,}32 \cdot 10^{-2} \cdot 10^{-1}}_{C} \cdot \underbrace{10}_{R}$$

$C = 0{,}32\ mF$

$R = 10\ \Omega$

NB la scelta di dimensionare C dell'ordine dei mF deriva dalle richieste di $f_T$ in Hz




$$A = -\frac{R_2}{R_1} \longrightarrow R_2 = R_1 = R$$